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REGULAR ARTICLE

Price informativeness and adaptive trading

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Abstract
We propose a structural model in which utility-maximizing investors strategically switch between the fundamental and chartist trading strategy in a market with asymmetric information. Their adaptive trading behavior generates an evolving price discovery process that reshapes the market environment, which then feeds back on their subsequent trading actions. The model implies that the price is more informative in terms of incorporating new information about the fundamental value when the asset is significantly mispriced and when the information is more precise. These theoretical predictions are supported by empirical evidence based on I/B/E/S data from January 2000 to December 2015.

Keywords Price informativeness · Trading heterogeneity · Market efficiency · Price discovery

JEL Classification G12 · D52 · D83

1 Introduction

The financial market provides an important function of price discovery. The market price aggregates information embedded in investors' trading activities. In an efficient market, new information is incorporated quickly into the price through the trading of rational investors. However, in reality, investors may have different perceptions of the same information (Harrison and Kreps 1978) and act selectively on the information that is more likely to be impounded into the price (Froot et al. 1992). As a result,

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the market price reflects the aggregation of opinions, which does not necessarily convey all information available. The fact that investors are not always rational and the presence of asymmetric information further complicate the information-revealing process. How information is transmitted into the market price via investors' complex trading behavior remains an open question.

This paper seeks to understand how investors' trading behavior responds to new information under a dynamic market environment, and how their collective actions shape price informativeness. We propose a structural model with heterogeneous and boundedly rational investors, who strategically switch between informed and uninformed trading to maximize their utilities in a market with asymmetric information. Due to information friction, investors form diverse perceptions on the attractiveness of relatively informed fundamental strategy, which considers all available information about the fundamentals and bets the price to reflect value, and the uninformed chartist strategy, which extrapolates price movements from historical trends only. As the market is not always efficient (Shiller 2003), investors in our model do not always act on the fundamental strategy as they may be better off switching away from it under certain circumstances. In other words, investors respond to information selectively. Therefore, in our model, whether the price is informative or not is the outcome of aggregate actions of all investors instead of pre-determined. The adaptive trading activities of investors shape the price discovery process and lead to various degrees of price informativeness, measured by the extent to which new information is incorporated into the price. The price is considered to be more informative if it is more responsive to new information. We focus on the price responsiveness to new information flows as opposed to the full information level. This is because old information may become stale or even obsolete as new information arrives (O'Hara 2003).

How much existing information has been reflected in the price is, however, crucial for the subsequent price to incorporate new information, as it affects the extent to which agents act on information. If the current price reflects little information about the fundamental value such that the asset is significantly mispriced, the potential gain for trading such an asset is large once the price incorporates all information. Our model implies that, when the asset is originally more significantly mispriced, more agents are motivated to adopt the fundamental strategy, which buys the asset when it is undervalued and sells it when it is overvalued. The trading actions of fundamentalists drive the price towards its fundamental value and helps incorporate information about the fundamental into the price. An increase in the number of fundamentalists that act on information improves the subsequent price informativeness. Such a finding is consistent with the efficient market hypothesis (EMH), which predicts stock prices to reflect all available information. Intuitively, when new information arrives, the market will not react exclusively to such news but also react to existing information that is not yet reflected in the price. As a result, the price appears to respond more sensitively to new events when less existing information has been incorporated into the price. In other words, greater mispricing today is associated with a more informative price in the future.

The information quality is another important factor for agents' adaptive trading behavior and the price discovery process. Our model shows that more precise information can either improve or undermine the price informativeness, depending on the relative trading power of heterogeneous agents. Jin and Myers (2006) find that the market price contains more private information in a more transparent environment when the information is less noisy (or more precise), while Dasgupta et al. (2010) show the opposite. Our model reconciles these two distinct results by showing a nonlinear relation between price informativeness and information quality that is contingent on the relative trading power of fundamentalists who act on their private information on fundamental value, and chartists whose trading behavior relies on historical price trends. Our model predicts that, more precise information always increases the market fraction of fundamentalist, as the reduction in information risk renders the fundamental strategy more attractive. While a larger number of fundamentalists generally leads to a more informative market price, there are exceptions. Our model also predicts that when market information is sufficiently precise, it is impossible for chartists to outweigh fundamentalists in their contribution to the price discovery. However, when information is relatively noisy, chartists may contribute more to price informativeness than fundamentalists under certain circumstances. Given that more precise information increases the fraction of fundamentalists, this implies that initial improvement in price quality could lead to a fall in price informativeness, but price informativeness will ultimately increase as information quality continues to improve.

The impact of asset mispricing and information quality on price informativeness is supported by empirical evidence based on data from the Institutional Brokers' Estimate System (I/B/E/S). As more precise information indicates lower information risk, we follow (Johnson 2004) and use the dispersion among analysts' forecasts as a proxy for information quality. As analysts act like fundamentalists in utilizing both private and public information to estimate the value of the asset (Cooper et al. 2001; Barber et al. 2010), each estimated price target proxies well the conditional expected fundamental value of the underlying asset by each fundamentalist. Under the assumption of uniformly distributed private signals, the Bayesian update in our setup suggests that estimated fundamental value is uniformly distributed around the true fundamental value. Therefore, we proxy the fundamental value by the median value of analysts' price targets. The asset mispricing is measured by the absolute difference between the market price and its fundamental value. Finally, price informativeness is measured by the degree to which the market price changes in response to updates of the fundamental value. The estimation results show that the market price is more informative when the asset is more significantly mispriced. As the condition for a negative relation between price informativeness and information precision is quite restrictive, our empirical results document the dominant effect of more precise information in increasing price informativeness.

Related literature In rational expectations equilibrium (REE) models, a popular framework to explore the implications of the market microstructure in price informativeness is where noise trading has no price impact, and noise traders always suffer losses. In the behavioral noise trader approach, however, uninformed or irrational trading can affect the market price and be profitable and, as a consequence, sustains their prevalence in the financial market (De Long et al. 1990; Shleifer and Summers 1990). This paper follows the latter approach to study the impact of heterogeneous and adaptive trading on price informativeness. It further argues that investors may choose to ignore market information and act irrationally since uninformed trading yields greater utility in certain market environments. Our study contributes to the understanding of how selective reaction to information based on adaptive trading shapes price informativeness.

Our model is characterized by investors who interact with each other and switch between the fundamental and chartist trading strategies to maximize their utilities. These heterogeneous agents are assumed to use either fundamental or technical analysis to form their expectations of future asset price movements. This setup is closely related to, inter alia, Day and Huang (1990), Lux (1995), Brock and Hommes (1998), and He and Westerhoff (2005), Huang et al. (2010) and Lof (2012). In these models, the fraction of fundamentalists and chartists is either fixed (Day and Huang 1990; Huang and Zheng 2012) or exogeneously determined by some random switching process (Lux 1995; Brock and Hommes 1998; He and Westerhoff 2005; Lof 2012). The hypothesis on evolutionary selection among different forecasting rules is supported by both experimental and empirical evidence (Boswijk et al. 2007; Anufriev and Hommes 2012; Lof 2015). This strand of literature highlights that the notion investors can switch between heterogeneous strategies, which implies that they understand both fundamental and technical analysis. Consistent with such an implication, each investor in our model forecasts the future price movements using both valuation methodologies and utilizing information drawn from them. This aspect of the model is closely related to the work of Barberis et al. (2018), who model investors' demand as a weighted average of both types of valuation. Following He and Zheng (2016), this paper adds another layer to this class of models by introducing information friction and endogenizing the fraction of fundamentalists in the market. We show that, in a market with incomplete information, investors who seek to maximize their utilities will follow either the fundamental or the chartist strategy adaptively. However, because of information dispersion, some investors expect the fundamental strategy to outperform the chartist strategy, while others believe the opposite. This leads to heterogeneous trading in the market. Changing market environments, i.e. asset price and information quality, affect investors' perceptions on the performance of heterogeneous strategies, which may motivate them to switch from one strategy to another. Based on such a framework, we add to current literature that focuses on simulation-based solutions in a complex dynamic system by documenting analytically the relation between price informativeness and adaptive heterogeneous trading activities. Moreover, these theoretical findings were well supported by empirical evidence drawn from the Institutional Brokers' Estimate System (I/B/E/S) that provide analysts forecasts on listed companies' value. The empirical evidence further supports the endogenized trading heterogneeity documented in Shi and Zheng (2018).

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 conducts a comparative analysis to understand the role of different market factors on the fraction of fundamentalists and price informativeness. Section 5 tests the theoretical predictions using I/B/E/S data. Section 6 concludes.

2 The model

We consider a continuum of [0, 1] agents (i.e. fund managers) trading on one risky and one risk-free asset in a market with incomplete information. The risk-free interest rate is normalized to 0. Each agent observes a private but noisy signal about the fundamental value. Conditional on their signals and the prevailing market environment, agents make trading decisions to maximize their utility. The trading of the risky asset will follow either a fundamental strategy, characterized by the expectation that the market price reflects its fundamental value, or a chartist strategy, which relies on extrapolation of past price trends to forecast the asset price. Agents evaluate the optimal utility for each strategy and place trading orders based on the strategy that yields a higher utility. We now turn to describe the model in detail.

2.1 Information structure

In each period t, the latest logarithmic fundamental value of the risky asset θ_t is realized but not revealed to the public. The prior of θ_t is governed by

$$\theta_t = \theta_{t-1} + e_t,$$

where e_t follows an improper uniform distribution. The log of historical fundamental value $\boldsymbol{\theta}^{t-1} = (\theta_{t-1}, \theta_{t-2}, \ldots, \theta_0)$ is public information for all investors at time t. Upon the realization of θ_t, each agent $i \in [0, 1]$ observes a private signal about the fundamental value, $x_{i,t} = \theta_t + \varepsilon_{i,t}$, where $\varepsilon_{i,t}$ is the information noise term that is independently and uniformly distributed over $[-\varepsilon_t, \varepsilon_t]$. Conditional on his private signal $x_{i,t}$, an agent i's posterior update of the mean and variance of the real fundamental value θ_t is

$$\begin{aligned} E\left(\theta_t | x_{i,t}\right) &= x_{i,t} \\ Var\left(\theta_t | x_{i,t}\right) &= \varepsilon_t^2. \end{aligned}$$

2.2 Asset allocation under heterogeneous strategies

All agents share a common mean-variance utility function[1]

$$U_{i,t+1} = y_{i,t} E_i(R_{t+1}) - \frac{\alpha}{2} y_{i,t}^2 Var_i(R_{t+1}), \tag{1}$$

where α is the absolute risk aversion coefficient, $E_i(R_t)$ and $Var_i(R_t)$ are agent i's expected mean and variance of the return on the risky asset, and $y_{i,t}$ is the amount of capital allocated to the risky asset at period t. Each agent seeks to maximize his

[1] Such a mean-variance utility function is equivalent to constant absolute risk aversion (CARA) utility function when agents wealth are normally distributed.

utility by allocating capital between the risky and the risk-free assets. Solving the optimization problem yields the optimal demand for the risky asset:

$$y_{i,t} = \frac{E_i(R_{t+1})}{\alpha Var_i(R_{t+1})}. \tag{2}$$

We assume that each agent is offered investment advice by his research team that analyzes investment opportunities based on two prevailing and complementary methods — fundamental and technical (or chartist) strategies. We focus on these two strategies for several reasons. First, they are commonly used in the financial industry and frequently observed in laboratory experiments (Anufriev and Hommes 2012). Second, in practice, it is costly to hire different researchers to conduct a wide variety of analysis, and therefore it makes sense to focus on the commonly used strategies due to resource constraint. Third, trading based on fundamental and chartist analysis is representative to capture key financial market patterns (Lux 1995; He and Westerhoff 2005; Huang et al. 2012).

Based on fundamental analysis, the price is expected to revert to its fundamental value so that an agent will buy (sell) the risky asset when the price is below (above) the fundamental value. Such a fundamental strategy is relatively informed because it utilizes all available information about the fundamental value. As the prior distribution of θ_t is uniform and the signal is also uniformly distributed around θ_t, agent i's posterior update of θ_t is uniformly distributed over the interval $\left[x_{i,t} - \varepsilon_t,\ x_{i,t} + \varepsilon_t\right]$. Let p_t be the logarithmic price of the risky asset at period t and $R_t = p_t - p_{t-1}$ be the return. The expected mean and variance of return in period $t+1$ based on fundamental analysis are as follows:

$$E_i^f(R_{t+1}) = E_i\left(\theta_{t+1}|x_{i,t}\right) - p_t = x_{i,t} - p_t, \tag{3}$$

$$Var_i^f(R_{t+1}) = Var_i\left(\theta_{t+1}|x_{i,t}\right) = \varepsilon_t^2. \tag{4}$$

In contrast, based on technical or chartist analysis, the expected price and variance of agent i are independent of her signal x_i but rely on historical price patterns such that

$$E_i^c(R_{t+1}) = \beta\,(p_t - v_t) \tag{5}$$

$$Var_i^c(R_{t+1}) = \beta^2\sigma^2, \tag{6}$$

where v_t is a reference price or a price trend that is derived from historical price movements of the asset based on some technical analysis[2], β measures the extrapolation of price deviations from the trend, and σ^2 is a heuristic expectation of the variance of the asset returns, which can be derived from the variance of historical returns. The reference price v_t can be a moving average, a supporting (resistance) price level, or any index derived from technical analysis. In particular, when v_t prices and $\beta > (<)0$, the strategy c is essentially a time-series momentum (contrarian) strategy. A positive value of $p_t - v_t$ indicates a bullish sentiment, while a negative value

[2]We keep the specification of v_t open to make the results more general. The results in this paper hold if v_t is i.e. a moving average of the past price such that $v_t = \sum_{i=1}^{i=n}\omega_i p_{t-i}$, where ω_i is the weight for the price i periods ago.

means a bearish sentiment. For now, we keep the specification of v_t open to keep the model general. Such a chartist strategy is uninformed as it ignores the information on the fundamental value.

Note from Eq. 2 that different perceptions about future price movements result in different optimal demands for the risky asset. Substituting (3) and Eq. 4 into Eq. 2 yields the optimal demand for the risky asset based on fundamental analysis

$$y_{i,t}^{f} = \frac{x_{i,t} - p_t}{\alpha \varepsilon_t^2}. \tag{7}$$

Information dispersion leads to diverse expectation of the fundamental value and therefore different demand for the risky asset, even if agents adopt the same fundamental strategy. Let $\kappa_t = |\theta_t - p_t|$ and $\kappa_{i,t} = \left|x_{i,t} - p_t\right|$ be the actual and expected degree of mispricing, respectively. The higher the value of κ_t, the greater the degree of mispricing and the less efficient the price is in incorporating existing information about the fundamental. When $\kappa_t = 0$, that is, $\theta_t = p_t$, the price fully reflects the fundamental value and all existing information about the fundamental is incorporated into the price. According to Eq. 7, the amount of capital moving in or out of the risky asset based on fundamental analysis ($\left|y_{i,t}^{f}\right|$) is an increasing function of one's expected degree of mispricing. It means that a transaction based on fundamental strategy is more aggressive when the price is less efficient in incorporating existing information.

Similarly, the optimal demand for the risky asset conditional on chartist analysis is obtained by substituting (5) and Eq. 6 into Eq. 2:

$$y_{i,t}^{c} = \frac{p_t - v_t}{\alpha \beta \sigma^2}. \tag{8}$$

The demand function based on chartist analysis is common for all agents. As trade based on Eq. 8 is uninformative about the fundamental value, it captures uninformed trading activities. Equations 7 and 8 corresponds to the "value" and the "growth" signals in Barberis et al. (2016).

2.3 Choosing between heterogeneous strategies

We follow He and Zheng (2016) to formalize the choice of trading strategies. In particular, we assume that each agent can only select one of the two investment strategies specified in Eqs. 7 and 8, which are grounded in fundamental and chartist analysis, respectively. The true data generating process of the asset price is hard to predict as there could be feedback effects coming from trading actions to asset prices that alter the original data generating process of the asset price. Agents understand that fundamental and chartist analysis are imperfect frameworks to predict stock price movements, but there is no evidence that other methods consistently and significantly outperform these two. Moreover, it takes time and resources to explore useful alternative valuation methods but trading requires quick actions. Due to the constraints of risk exposure, time and resources, they have no incentive to choose strategies other than the fundamental and the chartist strategies offered by their research teams.

Given the market complexity and uncertainty, there is no reliable rule of thumb to tell which strategy does a better job of forecasting future price movements. Against this backdrop, all agents treat the two strategies equally in terms of their forecasting precision. All agents consider themselves to be price takers and do not consider the impact of their trading on the asset price. We therefore assume that each agent simultaneously and independently chooses the strategy that yields a higher utility after observing the latest price and the private signal. Agents' choice of strategy can be summarized in the following two steps.

First, each agent calculates the maximum expected utility based on the fundamental and the chartist strategies, which can be obtained respectively by substituting (7) and Eq. 8 into Eq. 1:

$$E^f\left(U_{i,t+1}\right) = \frac{\left(x_{i,t} - p_t\right)^2}{2\alpha\varepsilon_t^2}, \tag{9}$$

$$E^c\left(U_{i,t+1}\right) = \frac{\left(p_t - v_t\right)^2}{2\alpha\sigma^2}, \tag{10}$$

where $E^f\left(U_{i,t+1}\right) = \max E_i^f\left(U_{i,t+1}\right)$ and $E^c\left(U_{i,t+1}\right) = \max E^c\left(U_{i,t+1}\right)$ are agent i's expected maximum utilities based on the fundamental and the chartist strategies.

Second, each agent compares the maximum expected utilities from the two strategies and selects the one with a higher expected utility. Let $\bar{x_t}$ be the threshold signal that makes an agent indifferent between the two strategies such that $E^f\left(U_{i,t+1}\right) = E^c\left(U_{i,+1}\right)$ when $x_{i,t} = \bar{x_t}$, that is,

$$\frac{(\bar{x_t} - p_t)^2}{2\alpha\varepsilon_t^2} = \frac{(p_t - v_t)^2}{2\alpha\sigma^2}$$

Solving for $\bar{x_t}$ yields

$${x_t}^{\pm} = \pm\frac{\varepsilon\left(p_t - v_t\right)}{\sigma} + p_t$$

Let x_t^m and x_t^M denote the two solutions of $\bar{x}$ such that

$$x_t^m = \min({x_t}^{\pm}) = -\frac{\varepsilon_t\left(p_t - v_t\right)}{\sigma} + p_t, \text{ and} \tag{11}$$

$$x_t^M = \max({x_t}^{\pm}) = \frac{\varepsilon_t\left(p_t - v_t\right)}{\sigma} + p_t. \tag{12}$$

When $E^f\left(U_{i,t+1}\right) = E^c\left(U_{i,t+1}\right)$, an agent is indifferent between the fundamental and the chartist strategies. In that case, we follow the conventional wisdom and assume that he will choose the fundamental strategy. Clearly, when $p_t = v_t$, all agents will choose the fundamental strategy as $E^f\left(U_{i,t+1}\right) \geq E^c\left(U_{i,t+1}\right) = 0$ for any i. Given that $E^f\left(U_{i,t+1}\right)$ is a nonlinear function of x_i while $E^c\left(U_{i,t+1}\right)$ is independent of $x_{i,t}$, as illustrated in Fig. 1, $E^f\left(U_{i,t+1}\right) \geq E^c\left(U_{i,t+1}\right)$ when $x_{i,t} \geq x_t^M$ or $x_{i,t} \leq x_t^M$, and $E^f\left(U_{i,t+1}\right) < E^c\left(U_{i,t+1}\right)$ otherwise. To put it differently, an agent will choose the chartist strategy if his signal falls into the interval (x_t^m, x_t^M) but will

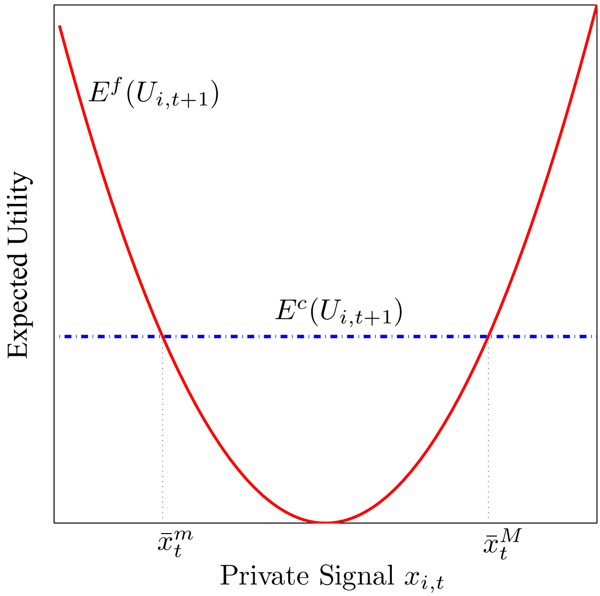

Fig. 1 Threshold signals. The solid and dashed lines plot the expected utility for fundamental and chartist strategy respectively. The two strategies yield the same utility if private signal $x_{i,t}$ falls on the thresholds x_t^m or x_t^M

choose the fundamental strategy otherwise. Therefore, the actual trading order placed by agent i is given by

$$y_{i,t} = \begin{cases} y_{i,t}^f & \text{if } x_{i,t} \le x_t^m \text{ or } x_{i,t} \ge x_t^M; \\ y_{i,t}^c & \text{if } \quad x_{i,t} \in (x_t^m,\ x_t^M) \end{cases}. \tag{13}$$

To facilitate our analysis below, agent i is considered to be a fundamentalist if $y_{i,t} = y_{i,t}^f$, and a chartist if $y_{i,t} = y_{i,t}^c$. Clearly, Eq. 13 indicates that an agent will become a fundamentalist if and only if he expects the degree of mispricing is sufficiently large such that $\kappa_{i,t} = \left|x_{i,t} - p_t\right| \ge \varepsilon_t \left|p_t - v_t\right| / \sigma$. The result suggests that agents whose signals fall in the tails of the distribution are more likely to favor the fundamental strategy.

2.4 Market fraction of fundamentalists

Changes in the asset price or the information structure shape the market fractions of fundamentalists and chartists. Holding all else constant, when the price changes, the two threshold signals x_t^m and x_t^M will also change, which in turn affects the number of agents whose signals fall into (x_t^m, x_t^M) and thus the market fraction of each type of agent. Changes in the fundamental value or the distribution of new signals have similar effects. We take into account these factors and study how investors switch between the two strategies in response to the dynamic market environment.

Denote m_t as the fraction of fundamentalists in the market. As the market consists of fundamentalists and chartists only, the fraction of chartists is then given by $1-m_t$. Since x_i is uniformly distributed over $[\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$ and chartists are agents whose signals fall into (x_t^m, x_t^M), we consider the fraction of fundamentalists under five scenarios.

First, if $\theta_t + \varepsilon_t < x_t^m$ or $\theta_t - \varepsilon_t > x_t^M$, then all agents' signals fall below x_t^m or above x_t^M, which suggests that all agents become fundamentalists according to Eq. 13 and $m_t = 1$. Second, if $\theta_t + \varepsilon_t < x_t^M$ and $\theta_t - \varepsilon_t > x_t^m$, then all signals fall into (x_t^m, x_t^M) so that all agents are chartists and $m_t = 0$. Third, if $x_t^m \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$ and $\theta_t + \varepsilon_t < x_t^M$, then agents whose signals fall into $\left[\theta_t - \varepsilon_t, x_t^m\right]$ become fundamentalists and $m_t = \dfrac{x_t^m - \theta_t + \varepsilon_t}{2\varepsilon_t}$. Fourth, if $x_t^M \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$ and $\theta_t - \varepsilon_t > x_t^m$, then agents whose signals fall into $\left[\theta_t - \varepsilon_t, x_t^M\right]$ become fundamentalists and $m_t = \dfrac{\theta_t + \varepsilon_t - x_t^M}{2\varepsilon_t}$. Finally, if $x_t^m, x_t^M \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$, then $m_t = \dfrac{2\varepsilon_t - x_t^M + x_t^m}{2\varepsilon_t}$. To summarize, the fraction of fundamentalists in the market is given by

$$m_t = \begin{cases} 1 & \text{if } \Omega_1 \\ 0 & \text{if } \Omega_2 \\ \frac{x_t^m - \theta_t + \varepsilon_t}{2\varepsilon_t} & \text{if } \Omega_3 \\ \frac{\theta_t + \varepsilon_t - x_t^M}{2\varepsilon_t} & \text{if } \Omega_4 \\ \frac{2\varepsilon_t - x_t^M + x_t^m}{2\varepsilon_t} & \text{if } \Omega_5 \end{cases} \tag{14}$$

$$\text{where } \begin{aligned} \Omega_1 &= \left\{x_t^m, x_t^M \mid \theta_t + \varepsilon_t < x_t^m \text{ or } \theta_t - \varepsilon_t > x_t^M\right\} \\ \Omega_2 &= \left\{x_t^m, x_t^M \mid \theta_t + \varepsilon_t < x_t^M \text{ and } \theta_t - \varepsilon_t > x_t^m\right\} \\ \Omega_3 &= \left\{x_t^m, x_t^M \mid x_t^m \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t] \text{ and } \theta_t + \varepsilon_t < x_t^M\right\} \\ \Omega_4 &= \left\{x_t^m, x_t^M \mid x_t^M \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t] \text{ and } \theta_t - \varepsilon_t > x_t^m\right\} \\ \Omega_5 &= \left\{x_t^m, x_t^M \mid x_t^m, x_t^M \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]\right\} \end{aligned}$$

As signals are uniformly distributed over $[\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$, conditional on Eq. 14, the average signal of fundamentalists is

$$\eta_t = \begin{cases} \theta_t & \text{if } \Omega_1 \\ 0 & \text{if } \Omega_2 \\ \frac{x_t^m + \theta_t - \varepsilon_t}{2} & \text{if } \Omega_3 \\ \frac{\theta_t + \varepsilon_t + x_t^M}{2} & \text{if } \Omega_4 \\ \frac{(x_t^m)^2 - (x_t^M)^2 + 4\varepsilon_t\theta_t}{2x_t^m - 2x_t^M + 4\varepsilon_t} & \text{if } \Omega_5 \end{cases}, \tag{15}$$

Given the average signal of fundamentalists, η_t, as specified in Eq. 15, the average demand for the risky assets by all fundamentalists, denoted as $\bar{y}^f$, can be calculated by replacing the individual signal $x_{i,t}$ in Eq. 7 with η_t so that

$$\bar{y}_t^{\,f} = \frac{\eta_t - p_t}{\alpha\varepsilon_t^2}. \tag{16}$$

2.5 Asset price formation

All agents, after selecting their strategies, submit trading orders to a market maker according to Eq. 13. Following Day and Huang (1990) and Lux (1995), we assume the market maker updates the price according to the aggregate demand D_t, the sum of all trading orders:

$$R_{t+1} = p_{t+1} - p_t = \gamma D_t, \tag{17}$$

where γ measures the marginal impact of the aggregate demand on the price change,[3] and D_t is given by

$$D_t = \sum_{i\in[0,1]} y_{i,t} = m_t \bar{y}_t^{\,f} + (1-m_t)\, y_t^c$$
$$= \begin{cases} \dfrac{\theta_t - p_t}{\alpha\varepsilon_t^2} & \text{if } \Omega_1 \\ \dfrac{p_t - v_t}{\alpha\beta\sigma^2} & \text{if } \Omega_2 \\ \dfrac{\frac{x_t^m+\theta_t-\varepsilon_t}{2} - p_t}{\alpha\varepsilon_t^2}\frac{x_t^m-\theta_t+\varepsilon_t}{2\varepsilon_t} + \dfrac{p_t - v_t}{\alpha\beta\sigma^2}\frac{\theta_t+\varepsilon_t-x_t^m}{2\varepsilon_t} & \text{if } \Omega_3 \\ \dfrac{\frac{\theta_t+\varepsilon_t+x_t^M}{2} - p_t}{\alpha\varepsilon_t^2}\frac{\theta_t+\varepsilon_t-x_t^M}{2\varepsilon_t} + \dfrac{p_t - v_t}{\alpha\beta\sigma_{t-1}^2}\frac{x_t^M-\theta_t+\varepsilon_t}{2\varepsilon_t} & \text{if } \Omega_4 \\ \dfrac{\frac{(x_t^m)^2-(x_t^M)^2+4\varepsilon_t\theta_t}{2x_t^m-2x_t^M+4\varepsilon_t} - p_t}{\alpha\varepsilon_t^2}\frac{2\varepsilon_t-x_t^M+x_t^m}{2\varepsilon_t} + \dfrac{p_t - v_t}{\alpha\beta\sigma_{t-1}^2}\frac{x_t^M-x_t^m}{2\varepsilon_t} & \text{if } \Omega_5 \end{cases}. \tag{18}$$

The second equality line is obtained by substituting m_t, $\bar{y}_t^{\,f}$ and y_t^c with Eqs. 14, 16, and 8 respectively and by substituting η_t in Eq. 16 with Eq. 15.

3 Comparative analysis

Agents' investment decisions critically depend on how much existing information has been incorporated into the price, how precise the private signals are, and how bullish or bearish the market sentiment, and their collective actions determine how efficient the subsequent price incorporates new information. In this section, we conduct comparative analysis to understand the dynamic interaction between agents trading behavior and asset price as well as its implications for price informativeness.

3.1 Market fraction of fundamentalists

The degree of mispricing, κ_t, captures how much existing information has been incorporated into the current price, which is an important measure of price efficiency. The imperfect information leads to heterogeneous expectation on the degree of mispricing, which shapes the market structure with different composition of fundamentalists

[3] Including the supply shock $S \sim N\left(0, \sigma_s^2\right)$ to the right hand side of Eq. 17 does not affect the main results.

and chartists. Lemma 3.1 summarizes the relation between the market fraction of fundamentalists, m_t, and the degree of mispricing, κ_t. Other than some specific circumstances, m_t increases with κ_t, which means that agents are more motivated to be fundamentalists if the current asset is more significantly mispriced. The market fraction of fundamentalists decreases with the degree of mispricing if and only if the fundamental and the chartist strategies suggest opposite trading directions (i.e. one is buying and the other is selling), the change in price efficiency originates from a change in price, and either one of the three conditions are satisfied: (i) both threshold signals x_t^m and x_t^M fall into $[\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$ (condition Ω_5), or (ii) the information noise is sufficiently large ($\varepsilon_t > \sigma$) and one and only one of x_t^m and x_t^M falls into $[\theta_t - \varepsilon_t, \theta_t + \varepsilon_t]$ (condition Ω_3 or Ω_4).

Lemma 3.1 *$\partial m_t / \partial \kappa_t \geq 0$ except when $\partial \left| p_t - v_t \right| / \partial \kappa_t = 1$ and either condition (i) Ω_5, (ii) $\varepsilon_t > \sigma$ and Ω_3, or (iii) $\varepsilon_t > \sigma$ and Ω_4, is satisfied.*

Proof see Appendix. □

Remarks The intuition can be gained by tracking the source of the change in κ_t, which may be triggered by a change in either θ_t or p_t. Given Ω_1 or Ω_2, the market is composed entirely of only one type of investors - fundamentalists in the former case and chartists in the latter. It follows that the market fraction of fundamentalists in these cases is not sensitive to the degree of mispricing. Therefore, the following discussions focus on the relation between the market fraction of fundamentalists and the degree of mispricing conditional on Ω_3, Ω_4 or Ω_5.

An increase in κ_t stemming from a change in θ_t makes the fundamental strategy more appealing to all agents regardless of their signals (through increasing its expected utility in Eq. 9) while having no impact on the expected utility of the chartist strategy (see Eq. 9). As more agents adopt the fundamental strategy in expectation of gaining higher utilities, the fraction of fundamentalists increases. Theoretically, the result is also straightforward. Conditional on Ω_5, m_t is independent of θ_t, which follows directly from Eq. 14. Conditional on Ω_3, as $\theta_t - p_t < 0$, a decline in θ_t is equivalent to an increase in the degree of mispricing κ_t. According to Eqs. 11 and 5, a decline in θ_t has no impact on the two threshold signals x_t^m and x_t^M so that it widens the interval $\left[\theta_t - \varepsilon_t, x_t^m\right]$. Recall that agents whose signals fall into $\left[\theta_t - \varepsilon_t, x_t^m\right]$ will become fundamentalists (see Eq. 14). This implies that, conditional on Ω_3, a decline in θ_t leads greater price inefficiency, which in turn increases the market fraction of fundamentalists, m_t. Similarly, conditional on Ω_4, an increase in θ_t is associated with an increase in the degree of mispricing κ_t, which widens the interval $\left[x_t^M, \theta_t + \varepsilon_t\right]$ and increases m_t.

An increase in the degree of mispricing resulting from a change in the price has a different impact on the market fraction of fundamentalists. We analyze it in the following two cases:

(i) When chartists contribute to reducing the degree of mispricing, we have $\partial |p_t - v_t| / \partial \kappa_t = -1$, which means that $|p_t - v_t|$ declines as the price inefficiency κ_t increases. In this case, an increase in κ_t increases the expected utility of the fundamental strategy while reducing that of the chartist strategy. As a consequence, the market fraction of fundamentalists increases for the fundamental strategy, which now becomes more attractive. Theoretically, the results can be analyzed from three scenarios that correspond, respectively, to condition Ω_3, Ω_4 and Ω_5. Conditional on Ω_3, as $\theta_t - p_t < 0$ and chartists enhance price efficiency, we have $p_t - v_t < 0$. Therefore, given Ω_3, an increase in κ_t that is due to a price increase leads to a greater x_t^m (see Eq. 11). This results in a widened interval $\left[\theta_t - \varepsilon_t, x_t^m\right]$ and a higher m_t. Similarly, conditional on Ω_4, as $\theta_t - p_t > 0$, we have $p_t - v_t > 0$. As a result, an increase in κ_t that originates from a price decline leads to a smaller x_t^M (see Eq. 12), causing the interval $\left[x_t^M, \theta_t + \varepsilon_t\right]$ to widen and m_t to rise. Conditional on Ω_5, it is straightforward to see that an increase in κ_t is associated with a decline in $|p_t - v_t|$, which shrinks the interval $\left[x_t^m, x_t^M\right]$ and increases m_t (recall that $m_t = 1 - \frac{x_t^M - x_t^m}{2\varepsilon_t}$ conditional on Ω_5, see Eq. 14).

(ii) When chartists enlarge the degree of mispricing, $\partial |p_t - v_t| / \partial \kappa_t = 1$, which means$|p_t - v_t|$ increases with κ_t. Therefore, an increase in the degree of mispricing upgrades the expected utility of both strategies. Conditional on Ω_3 or Ω_4, when $\varepsilon_t \leq \sigma$, a greater degree of mispricing gives a bigger boost to the expected utility of the fundamental strategy relative to the chartist strategy, which improves the attractiveness of fundamental strategy and increases m_t. Similarly, when $\varepsilon_t > \sigma$, the expected utility of fundamental strategy increases by a smaller magnitude than that of the chartist strategy, which leads to a drop in m. Condition Ω_5 implies that $\theta_t - p_t \in [-\varepsilon_t, \varepsilon_t]$. Given that signals are uniformly distributed around θ_t, $x_i - p_t$ can be either positive or negative, which means that, while some agents expect the asset to be overpriced, others expect it to be underpriced. In this case, greater price inefficiency κ_t increases the expected utility of the chartist strategy for all agents as $\partial |p_t - v_t| / \partial \kappa_t = 1$. However, a bigger κ_t only increases the expected utility of the fundamental strategy for some agents while reducing that for others. As a result, the fraction of fundamentalists m_t declines. Theoretically, conditional on Ω_5, an increase in the degree of mispricing κ_t leads to an increase in $|p_t - v_t|$, which widens the interval $\left[x_t^m, x_t^M\right]$ and decreases m_t.

Agents trading decisions are also affected by information precision, which has an impact on agents expected price efficiency as well as risk. Lemma 3.2 suggests that the market fraction of fundamentalists increases with information precision.

Lemma 3.2 $\partial m_t / \partial \varepsilon_t \leq 0$, *where the equality holds if and only if the condition* Ω_1, Ω_2 *or* Ω_5 *is satisfied.*

Proof see Appendix. □

Remarks Intuitively, more precise information increases the expected utility of the fundamental strategy through reducing its expected risk, but does not affect the expected utility of the chartist strategy. Therefore, ceteris paribus, more information transparency enhances the attractiveness of the fundamental strategy relative to the chartist strategy, motivating more agents to adopt the former strategy.

The result can also be interpreted from another perspective. Recall that an agent will become a fundamentalist if and only if his expected price inefficiency, $\kappa_{i,t} = \left|x_{i,t} - p_t\right|$, exceeds the threshold value $\varepsilon_t \left|p_t - v_t\right| / \sigma$, which is an increasing function of the information noise ε_t. The more precise the information or, equivalently, the smaller the noise ε_t, the lower the minimum degree of expected price inefficiency that is sufficient to motivate fundamental trading. Holding all else constant, as information becomes more precise, the number of agents who expect the price inefficiency to exceed the threshold value $\varepsilon_t \left|p_t - v_t\right| / \sigma$ increases, which leads to a greater market fraction of fundamentalists, m_t.

3.2 Incorporation of new information

How current price impacts existing information is an important concept for price efficiency. However, how the future price is going to react to new information is perhaps more relevant as the insight allows investors to spot profit opportunities straightforwardly. The comparative analysis on the market fraction of fundamentalists enables us to explore how future price incorporates new information on the fundamental value in different market environment, which is termed price informativeness hereafter. Price informativeness is measured by $\partial p_{t+1}/\partial\theta_t$, the price sensitivity to a fundamental shock. By definition, the greater the value of $\partial p_{t+1}/\partial\theta_t$, the more efficient the price incorporates new information, that is, the more informative the price. In this section, we first examine the sign and magnitude of the price informativeness measured by $\partial p_{t+1}/\partial\theta_t$ and then analyze the relationship between price informativeness and various market factors.

3.2.1 Price informativeness

Proposition 1 suggests that, while the price generally responds positively to the fundamental shock, it can be misleading when the information noise is relatively large ($\varepsilon_t > \beta\sigma$) and chartists are trading more aggressively than fundamentalists to reduce mispricing ($\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ and $y_t^c\left(\theta_t - p_t\right) > 0$). The condition for $\partial p_{t+1}/\partial\theta_t < 0$ is valid if and only if $\varepsilon_t > \beta\sigma$. Proposition 1 also suggests that the price is always informative if the information is sufficiently precise such that $\varepsilon_t \le \beta\sigma$.

Proposition 1 $\partial p_{t+1}/\partial\theta_t \ge 0$ *except when* $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, $y_t^c\left(\theta_t - p_t\right) > 0$ *and* $\varepsilon_t > \beta\sigma$.

Proof see Appendix. □

Remarks We first explain the intuition behind the relatively rare scenario that$\partial p_{t+1}/\partial\theta_t < 0$ by looking into the comparative trading power (measured by the absolute demand) of fundamentalists and chartists. The condition $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ means that the trading power of chartists exceeds that of any fundamentalist. Note that the largest trading power of fundamentalists is $\frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}$, which is less than $\left|y_t^c\right|$. The condition $y_t^c\left(\theta_t - p_t\right) > 0$ implies that chartists are selling the risky asset when it is overpriced ($\theta_t < p_t$) and buying it when it is underpriced ($\theta_t > p_t$), contributing to a reduction in the magnitude of mispricing. Essentially, the condition for the price to respond negatively to a fundamental shock is that every chartist is selling (buying) more risky assets than any fundamentalist when the asset is overpriced (underpriced). In other words, chartists contribute more to the mitigation of mispricing than fundamentalists. Consider a case when the asset is overpriced. A positive fundamental shock $\Delta\theta_t$ raises the fundamental value θ_t and reduces the expected degree of mispricing, κ_t, leading to a decrease in the fraction of fundamentalists, m_t, according to Lemma 3.1. Note that $\partial m_t/\partial\kappa_t > 0$ when the change in κ_t is due to a fundamental shock. It is equivalent to shifting fundamentalists whose signals fall into $[\theta_t - \varepsilon_t, \theta_t + \Delta\theta_t - \varepsilon_t]$ to chartists after the shock. As chartists are selling more aggressively than any fundamentalist, such a switch of strategy strengthens the aggregate selling force, triggering a decline in the aggregate demand D_t and the subsequent drop in the asset price (see Eq. 17). To summarize, the price declines in response to a positive fundamental shock when $\theta_t < p_t$ and $-y_t^c \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$. The case when the asset is underpriced can be analyzed in a similar fashion.

If $\left|y_t^c\right| \notin \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ or $y_t^c\left(\theta_t - p_t\right) \leq 0$, then $\partial p_{t+1}/\partial\theta_t \geq 0$, which is expected to occur more frequently given the relatively loose constraints. The intuition behind $\partial p_t/\partial\theta_t \geq 0$ is straightforward. When $\theta_t < p_t$, a positive fundamental shock motivates agents to switch from fundamentalists to chartists, who are either buying ($y_t^c\left(\theta_t - p_t\right) \leq 0$) or selling less aggressively ($\left|y_t^c\right| \notin \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$) than fundamentalists. Whereas when $\theta_t \geq p_t$, some chartists switch to be fundamentalists, whose demand for the risky asset is not less than that of chartists. As a result, the aggregate demand and therefore the price of the risky asset increases after the fundamental shock, which suggests that the price is informative. When $\partial p_{t+1}/\partial\theta_t \geq 0$, we further evaluate whether the price overreacts or underreacts to the fundamental shock in the Appendix.

3.2.2 Overreaction

We next study whether the price overreacts or underreacts to a fundamental shock when $\partial p_{t+1}/\partial\theta_t > 0$. Given Ω_1, all agents are fundamentalists and respond fairly to the fundamental value of the risky asset such that $\partial p_{t+1}/\partial\theta_t = \gamma/\left(\alpha\varepsilon_t^2\right)$. We treat this case as the benchmark. The price is said to overreact (underreact) to a fundamental shock if $\partial p_{t+1}/\partial\theta_t > (<)\gamma/\left(\alpha\varepsilon_t^2\right)$. Proposition 2 highlights the environments that lead to overreaction to a fundamental shock. When (i) chartists' trading activities enlarge the degree of mispricing ($y_t^c(\theta_t - p_t) < 0$) and their trading power is sufficiently large ($\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$), or (ii) chartists trading activities mitigate the degree of mispricing ($y_t^c(\theta_t - p_t) > 0$) and their trading power is moderate ($\left|y_t^c\right| \in \left(\frac{\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2}\right)\right)$), the price responds more aggressively to a fundamental shock than the benchmark scenario when the market is composed entirely of fundamentalists.

Proposition 2 $\partial p_{t+1}/\partial\theta_t > \gamma/\left(\alpha\varepsilon_t^2\right)$ *if (i)* $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, $y^c(\theta_t - p_t) < 0$ *and* $\kappa_t > \max(0, \frac{\varepsilon_t(\beta\sigma-\varepsilon_t)}{\beta\sigma+\varepsilon_t})$, *or* $\left|y_t^c\right| \in \left(\frac{\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2}\right)\right)$, $y_t^c(\theta_t - p_t) > 0$, $\kappa_t > \varepsilon_t$ *and* $\varepsilon_t < \beta\sigma$; *and* $\partial p_{t+1}/\partial\theta_t \leq \gamma/\left(\alpha\varepsilon_t^2\right)$ *otherwise.*

Proof see Appendix. □

Remarks We first explain why condition (i) in Proposition 2 leads to an overreaction to the fundamental shock. The condition $y_t^c(\theta_t - p_t) < 0$ implies that chartists magnify the degree of mispricing by buying the risky asset when the asset is overpriced ($\theta_t < p_t$) and selling it when it is underpriced ($\theta_t > p_t$). Together with the condition $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, these two conditions mean that chartists buy (sell) a greater amount of the risky asset than that is sold (bought) by any fundamentalist when the asset is overpriced (underpriced). Consider a case when the asset is overpriced, such that $\theta_t < p_t$. The condition $y_t^c(\theta_t - p_t) < 0$ suggests that chartists will buy the risky asset when $\theta_t < p_t$, which enlarges the degree of mispricing. If all agents act as fundamentalists, an arbitrarily small positive fundamental shock $\Delta\theta_t$ would increase the aggregate demand by $\Delta\theta_t/\left(\alpha\varepsilon_t^2\right)$ (see Eq. 18). In the presence of chartists, such a shock reduces the degree of mispricing, κ_t, and decreases m_t by $\Delta\theta_t/(2\varepsilon_t)$ according to Lemma 3.1. Recall from Eq. 13 that the impact of a fundamental shock is equivalent to shifting agents whose signals fall into $(\theta_t - \varepsilon_t, \theta_t + \Delta\theta_t - \varepsilon_t)$ from fundamentalists to chartists. Note that the maximum individual demand of these agents is less than $\frac{\theta_t+\Delta\theta_t-p_t-\varepsilon_t}{\alpha\varepsilon_t^2}$ before the shock (the demand of a fundamentalist increases with his signal, see Eq. 7). After the shock, the demand of each of these agents increases to y_t^c, where $y_t^c > \frac{\theta_t+\Delta\theta_t-p_t+\varepsilon_t}{\alpha\varepsilon_t^2}$ according to condition (i). So, for each agent who switches from being a fundamentalist to being a chartist, the demand for the risky asset increases by more than $\frac{\theta_t+\Delta\theta_t-p_t+\varepsilon_t}{\alpha\varepsilon_t^2} - \frac{\theta_t+\Delta\theta_t-p_t-\varepsilon_t}{\alpha\varepsilon_t^2} = \frac{2}{\alpha\varepsilon_t}$. Overall, such switching behavior increases the aggregate demand by more than $\Delta\theta_t/\left(\alpha\varepsilon_t^2\right)$, which leads to an overreaction to the fundamental shock when the asset is overpriced. Similarly, we can show that the price overreacts to the fundamental shock when the asset is underpriced, as long as condition (i) is satisfied.

Condition (ii) in Proposition 2 means that chartists sell (buy) less aggressively than any fundamentalists when the asset is sufficiently overpriced (underpriced). The conditions $\kappa_t > \varepsilon_t$ and $\varepsilon_t < \beta\sigma$ ensure that $\left|y_t^c\right| \in \left(\frac{\kappa_t - \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2}\right)\right)$ and $y_t^c\left(\theta_t - p_t\right) > 0$ exist. Again, to explain the intuition, we consider a case when the asset is overpriced. In this case, condition (ii) implies that both fundamentalists and chartists sell the risky asset, and that chartists sell less aggressively than any fundamentalist such that $y_t^c > \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2}$. For each agent who switches from the fundamental to the chartist strategy, his demand increases by at least $2/\left(\alpha\varepsilon_t\right)$. A positive fundamental shock reduces the degree of mispricing κ_t, which decreases m_t by $\Delta\theta_t/\left(2\varepsilon_t\right)$ according to Lemma 3.1. Such a shock increases the aggregate demand by at least $\Delta\theta_t/\left(\alpha\varepsilon_t^2\right)$, which is greater than the increase under the benchmark scenario when all agents are fundamentalists. Such overreaction occurs regardless of whether the asset is initially overpriced or underpriced. It can either fuel the bubble or accelerate price recovery from distress.

To summarize, condition (i) or (ii) provides the necessary condition for each agent to increase his demand by at least $2/\left(\alpha\varepsilon_t\right)$ through switching from the fundamental to the chartist strategy, which leads to overreaction compared to the benchmark scenario when the market is composed entirely of fundamentalists. In all other scenarios not captured by condition (i) or (ii), each switching agent may either increase the demand by less than $2/\left(\alpha\varepsilon_t\right)$ or decrease the demand. A positive fundamental shock that reduces the market fraction of fundamentalists by $\Delta\theta_t/\left(2\varepsilon_t\right)$ affects the aggregate demand by less than $\Delta\theta_t/\left(\alpha\varepsilon_t^2\right)$, which leads to underreaction.

3.2.3 The impact of mispricing and information precision on price informativeness

How much information has been incorporated into the current price affects agents trading behavior which in turn determines how the subsequent asset price responds to new information. Proposition 3 suggests that the price is more informative when the asset is more significantly mispriced, that is, when κ_t is larger. It implies that price respond more sensitively to new information when less existing information was incorporated into current price.

Proposition 3 $\dfrac{\partial}{\partial\kappa_t}\dfrac{\partial p_{t+1}}{\partial\theta_t} \geq 0$, *where the equality holds if and only if* Ω_1, Ω_2 *or* Ω_5 *is satisfied.*

Proof see Appendix. □

Remarks To shed light on these results, consider two scenarios, with one being more mispriced than the other. We assume that these two scenarios only differ in the degree of mispricing. When the asset is overpriced, a positive fundamental shock $\Delta\theta_t$ leads to a reduction in the degree of mispricing κ_t and therefore a decline in the fraction of fundamentalists m_t under both scenarios (see Lemma 3.1). Recall from Eq. 13 that the impact of a fundamental shock of size $\Delta\theta_t$ is equivalent to switching agents whose signals fall into $\left(\theta_t - \varepsilon_t, \theta_t + \Delta\theta_t - \varepsilon_t\right)$ from fundamentalists to chartists. Among agents who switch strategy, their demand for the risky asset before the switch is smaller under the more mispriced scenario (see Eq. 7), but their demand after the switch are the same in both scenarios (because the chartist strategy yields the same demand in both scenarios). As a result, the switch from the fundamental to chartist strategy generates a greater increase in the demand for the risky asset in the more mispriced scenario. Therefore, the price will increase more in response to the positive fundamental shock, implying that the price is more informative when the asset is more significantly mispriced. Similar results can be obtained when the asset is underpriced.

The information environment also plays an important role on the incorporation of new information. Proposition 4 suggests that the price generally becomes more informative as information becomes more precise (i.e.ε_t decreases). However, when chartists trade sufficiently more aggressively than fundamentalists in enhancing market liquidity , that is, $\left|y_t^c\right| \in \left(\max\left(\frac{3\kappa_t+2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ and $y_t^c\left(\theta_t-p_t\right) > 0$, increasing information precision may reduce price informativeness. The condition (i) $\varepsilon_t \geq 3\beta\sigma$, or (ii) $\varepsilon_t \in (2\beta\sigma, 3\beta\sigma)$ and $\kappa_t < \frac{(\varepsilon_t-2\beta\sigma)\varepsilon_t}{3\beta\sigma-\varepsilon_t}$ ensure the existence of such a y_t^c.

Proposition 4 $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t} \leq 0$ *except when* $\left|y_t^c\right| \in \left(\max\left(\frac{3\kappa_t+2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, $y_t^c\left(\theta_t-p_t\right) > 0$ *and either (i)* $\varepsilon_t \geq 3\beta\sigma$, *or (ii)* $\varepsilon_t \in (2\beta\sigma, 3\beta\sigma)$ *and* $\kappa_t < \frac{\beta\sigma\varepsilon_t}{3\beta\sigma-\varepsilon_t}$.

Proof see Appendix. □

Remarks We first explain the rationale for why an improvement in information precision reduces price informativeness when $\left|y_t^c\right| \in \left(\max\left(\frac{3\kappa_t+2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$. Considering a case when $\theta_t < p_t$. As $y_t^c\left(\theta_t-p_t\right) > 0$, the constraint on $\left|y_t^c\right|$ implies that when $\theta_t < p_t$, $y_t^c < \min\left(\frac{-3\kappa_t-2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{-|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right)$, which means chartists sell more aggressively than any fundamentalist when the asset is overpriced. As discussed in Proposition 1, when there is a fundamental shock, agents switch from the fundamental to the chartist strategy, which decreases the aggregate demand and lowers the price. Recall that the impact of a positive fundamental shock is equivalent to shifting those whose signals fall into $(\theta_t-\varepsilon_t, \theta_t+\Delta\theta_t-\varepsilon_t)$ from fundamentalists to chartists. In response to the same fundamental shock, there will be more agents switching from the fundamental to the chartist strategy in a more transparent environment (the proportion of agents whot switch from fundamentalists to chartists is $\frac{\Delta\theta_t}{2\varepsilon_t}$). Before the switch, the demand of the fundamentalists who will later switch strategies is greater when information is more transparent (i.e. ε_t is smaller). After the switch, the demand of these agents is the same regardless of the information environment (because they all switch to chartists and all chartists share the same demand). It means that the switch from the fundamental to the chartist strategy reduces the demand by a greater amount when the information is more transparent. Therefore, the reduction in aggregate demand is greater when the information is more transparent. As a result, conditional on $y_t^c < \min\left(\frac{-3\kappa_t - 2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{-|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right)$ and $y_t^c(\theta_t - p_t) > 0$, the price decreases more in response to a positive fundamental shock when $\theta_t < p_t$, that is, $\partial p_{t+1}/\partial\theta_t$ becomes more negative in a more transparent environment when the asset is overpriced. Similarly, we can show that the price is less informative when $\theta_t > p_t$, as long as conditions outlined in Proposition 4 are satisfied.

When $\left|y_t^c\right| \notin \left(\max\left(\frac{3\kappa_t + 2\varepsilon_t}{\alpha\varepsilon_t^2}, \frac{|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right), \frac{\kappa_t + \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ or $y_t^c(\theta_t - p_t) \le 0$, then $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t} \le 0$. This is because the demand of agents who switch strategies in response to a fundamental shock increases by a larger magnitude (or decreases by a smaller magnitude) when the information is more precise. As a result, the aggregate demand is more positive (or less negative), and therefore price responds more sensitively to the fundamental shock, in a more transparent information environment.

It is worth pointing out that conditions for the price to be less informative in a more transparent environment are quite stringent. Moreover, these conditions can only be satisfied if the information is sufficiently noisy ((i) $\varepsilon_t \ge 3\beta\sigma$, or (ii) $\varepsilon_t \in (2\beta\sigma, 3\beta\sigma)$ and $\kappa_t < \frac{\beta\sigma\varepsilon_t}{3\beta\sigma - \varepsilon_t}$). Therefore, we shall expect the opposite scenario, that is, more informative price in a more transparent environment, to occur more frequently.

4 Empirical evidence

4.1 Data and proxies for variables

We test the theoretical implications using data from the Institutional Brokers' Estimate System (I/B/E/S), which collects analysts earnings estimates for listed companies. We require firms in our sample to be covered by at least five analysts for at least 24 months. The final sample includes 4754 distinct firms listed on the U.S. stock market, with a total of 340,485 firm-month observations from January 2000 to December 2015.

Most analysts track the cash flow of a company and forecast its future performance based on both private and public information, making them an ideal representation of practitioners of fundamental analysis in the real world. In fact, empirical studies find evidence that analysts play a similar role as fundamentalists in our model - they collect private information (Barber et al. 2010) and interpret the public information with their comparative advantage to discover the value of the stock (Cooper et al. 2001). We therefore interpret analysts' estimated prices as their best assessments of the fundamental value of the underlying asset. Assuming a uniform distribution in analysts' estimation, the fundamental value θ_t can be measured by the median of all analysts' forecast prices in logarithm. Following Chordia et al. (2006), we employ analysts' forecast dispersion to measure the information dispersion about the fundamental value. In particular, information noise is proxied by the logarithmic difference between the maximum and the minimum target prices. Following the model specification, the degree of mispricing, the absolute difference between the asset price and its fundamental value, is measured by the difference between the observed price and the median of the estimated target prices. The measure of price informativeness $\partial p_{t+1}/\partial\theta_t$ is proxied by $\Delta p_{t+1}/\Delta\theta_t$, that is, $r_t/\Delta\theta_t$ in a discrete time horizon, where $\Delta\theta_t$ is the innovation in the fundamental value.

Table 1 presents the summary statistics of the key variables. The information noise ε_t has an average value of 21%, which implies a significant divergence in analysts' estimation of fundamental value. The mean of the degree of mispricing, κ_t, is 49%, suggesting that the price on average fluctuates 49% above or below the fundamental value. The monthly return, r_t, and the innovation of the fundamental value, $\Delta\theta_t$, are both slightly negative over the sample period. However, the standard deviation of r_t is about 1.5 times as much as that of $\Delta\theta_t$, suggesting that the price is more volatile than the fundamental value, which is consistent with the excess volatility phenomenon.

4.2 Hypothesis development

Hypothesis 1: The price is informative.

The first hypothesis follows from Proposition 1. To test whether the price is informative, we estimate the coefficient Λ_1 in the following regression:

$$r_t = \Lambda_1\Delta\theta_t + \Upsilon X_t + e_{1,t},$$

where a higher Λ_1 stands for greater price informativeness, X_t is the set of control variables, and Υ is the coefficient matrix. If Λ_1 is positive and statistically significant, we cannot reject the null hypothesis that the price is informative.

Hypothesis 2: The price is more informative when the information is more precise.

Table 1 Summary statistics

	Mean	Standard deviation	Median	Minimum	Maximum	Observations
p_t	3.184	0.933	3.227	– 3.912	17.198	340485
θ_t	3.370	0.893	3.401	– 2.590	17.558	340485
ε_t	0.206	0.226	0.145	0.000	6.856	340485
κ_t	0.485	0.381	0.383	0.000	9.680	340485
r_t	– 0.002	0.143	0.009	– 4.191	4.300	340485
$\Delta\theta_t$	– 0.001	0.101	0.000	– 3.961	4.164	340485

The sample period is from January 2000 to December 2015. The variables p_t and θ_t are the logarithmic price and fundamental value of the stocks, κ_t is the degree of mispricing measured by the absolute value of the difference between p_t and θ_t, ε_t is the information noise measured by the log difference between the maximum and the minimum target prices, $\Delta\theta_t$ is the innovation in θ_t, and r_t is the stock return measured by the first-order difference of p_t

According to Proposition 4, price informativeness can either increase or decrease with information precision depending on the relative trading power of fundamentalists and chartists as well as the degree of information precision. To test the second hypothesis, we expand the regression above by adding the interaction between $\Delta\theta_t$ and the noise measure ε_t such that

$$r_t = \Lambda_1 \Delta\theta_t + \Lambda_2 \left(\Delta\theta_t \cdot \varepsilon_t\right) + \Upsilon X_t + e_{2,t},$$

where Λ_2 is the coefficient of the interaction term. A statistically significant and negative (positive) estimate of Λ_2 supports the hypothesis that the degree of price informativeness increases (decreases) with information precision. As the conditions for price informativeness to decrease with information precision are quite restrictive, the positive relation between price informativeness and information precision is expected to dominate.

Hypothesis 3: ***The price is more informative when existing mispricing is more severe.***

The third hypothesis follows directly from Proposition 3. In the same vein, to test whether the price is more informative when the asset is more significantly mispriced, we estimate the following equation:

$$r_t = \Lambda_1 \Delta\theta_t + \Lambda_3 \left(\Delta\theta_t \cdot \kappa_t\right) + \Upsilon X_t + e_{3,t},$$

where Λ_3 is the coefficient of the interaction between the fundamental innovation $\Delta\theta_t$ and the degree of mispricing κ_t. If the estimated coefficient Λ_3 is statistically significant and positive (negative), then one cannot reject the null hypothesis that price is more (less) informative when less existing information has been incorporated into the price. We expect the positive relation between price informativeness and the degree of mispricing to dominate because the conditions for it to hold are relatively loose compared to the alternative scenario.

4.3 Estimation results

Table 2 presents the estimation results. All regressions control for firm-level fixed effects. The estimated coefficient of $\Delta\theta_t$ reported in column 1 is statistically significant and positive, which supports ***Hypothesis 1*** (implied by Proposition 1) that the price is informative. Raising the fundamental value by 10% increases the return by 5.1%. There is no one-to-one relation between the change in the fundamental value and the asset price. But the positive coefficient indicates positive comovement between the asset price and the fundamental value.

Column 2 reports the results of the estimation that includes the interaction between the fundamental innovation $\Delta\theta_t$ and information noise ε_t. The estimated coefficient of the interaction term is negative and statistically significant, which means the return is less responsive to the fundamental innovation $\Delta\theta_t$ when the information is more opaque. In other words, price is more informative when the information is more precise, which supports ***Hypothesis 2***. It suggests that the positive relation between price informativeness and information precision dominates. The result is also intuitive, as investors are more confident to act on information with greater precision, which helps incorporate information into the price more efficiently.

Column 3 reports the estimation results that account for the interaction between the fundamental innovation, $\Delta\theta_t$, and the degree of mispricing, κ_t. The coefficient of the interaction term is positive and statistically significant, suggesting that price is more informative when the asset is originally more significantly mispriced, which supports ***Hypothesis 3*** (implied by Proposition 3). In particular, a 10% increase in the magnitude of mispricing raises the sensitivity of the return to fundamental innovation, $\Delta\theta_t$, by 1.3 percentage point. Noted that greater mispricing means more existing information fails to be incorporated into the price. When there is a fundamental shock, the price will not only respond to the new information but also the existing information, which magnifies the price sensitivity to the fundamental shock.

Column 4 reports the results that include both interaction terms. Column 5 reports the results from a similar regression as Column 4 but controls for not only firm but also year fixed effects. The finding that the price is more informative when the information is more precise and the asset is more significantly mispriced remains robust. For robustness checks, we proxy the actual fundamental value and the information

Table 2 Estimation results: impact of market elements on price informativeness

	(1)	(2)	(3)	(4)	(5)
$\Delta\theta_t$	0.511***	0.564***	0.341***	0.391***	0.388***
	(0.002)	(0.004)	(0.003)	(0.004)	(0.004)
$\Delta\theta_t \cdot \varepsilon_t$		– 0.057***		– 0.059***	– 0.061***
		(0.003)		(0.003)	(0.003)
$\Delta\theta_t \cdot \kappa_t$			0.131***	0.215***	0.212***
			(0.006)	(0.006)	(0.006)
ε_t		– 0.012***		0.046***	0.039***
		(0.001)		(0.001)	(0.001)
κ_t			– 0.326***	– 0.345***	– 0.362***
			(0.001)	(0.001)	(0.001)
Constant	– 0.002***	0.004***	0.066***	0.047***	0.098***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Firm fixed effect	Yes	Yes	Yes	Yes	Yes
Year fixed effect	No	No	No	No	Yes
Observations	340,485	340,485	340,485	340,485	340,485
R-squared	0.154	0.155	0.319	0.329	0.339

The dependent variable is the monthly asset return, r_t, calculated as the log difference of the price. The fundamental shock, $\Delta\theta_t$, is the innovation in the median target price reported by analysts covering the stock, ε_t is the information noise measured by the log difference between the maximum and the minimum target prices, κ_t is the market illiquidity measured by the absolute value of the log difference between the price and the median target price. All regressions control for firm fixed effect. Standard errors are reported in parentheses. Significance levels at 1, 5 and 10 percentage points are indicated, respectively, by ***, **, and * noise by the mean and the standard deviation of the analysts' target prices, calculate robust and/or clustered standard errors. In the results not reported, our key results that the price is informative and that price informativeness improves with the degree of mispricing and information precision remain robust.

5 Conclusion

We propose a structural model with endogenized market fraction of fundamentalists and chartists to understand the determinants of price informativeness. In a market with asymmetric information, utility-maximizing investors strategically switch between heterogeneous trading strategies, accounting for dynamic market conditions. Such utility-maximizing behavior motivates them to react to information selectively and to place trading orders adaptively. Their trading behavior incorporates new information regarding the fundamental value in an evolutionary and adaptive process. This paper shows that greater degree of mispricing and more precise information enhances future price informativeness by motivating more investors to adopt a fundamental strategy that responds positively to innovation in the fundamental value as well as existing information that has not yet been incorporated into price, which consistently contribute to improve price discovery. These theoretical predictions are supported by empirical evidence.

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Compliance with Ethical Standards

Conflict of interests The authors declare that they have no conflict of interest.

Appendix

Proof

Proof of Lemma 3.1

Proof Differentiating m_t with respect to the degree of mispricing κ_t yields

$$\frac{\partial m_t}{\partial \kappa_t} = \begin{cases} 0 & \text{if } \Omega_1 \text{ or } \Omega_2 \\ \dfrac{1}{2\varepsilon_t} - \dfrac{1}{2\sigma}\dfrac{\partial\,|p_t - v_t|}{\partial \kappa_t} & \text{if } \Omega_3 \text{ or } \Omega_4 \\ -\dfrac{1}{\sigma}\dfrac{\partial\,|p_t - v_t|}{\partial \kappa_t} & \text{if } \quad \Omega_5 \end{cases}.$$

A marginal change in $\kappa_t = |\theta_t - p_t|$ can originate from a change in the fundamental value θ_t or the price p_t. If an increase in market illiquidity comes from a change in θ_t, then $\partial |p_t - v_t| / \partial \kappa_t = 0$, which leads to $\partial m_t / \partial \kappa_t > 0$ if Ω_3 or Ω_4 and $\partial m / \partial \kappa_t = 0$ if Ω_5.

If the change in κ_t is due to a change in p_t, the relation between m_t and κ_t depends on the trading directions of the fundamental and the chartist strategies as well as the information noise. When chartists improve market liquidity, i.e. $p_t - v_t > 0$ and $\kappa_t = \theta_t - p_t > 0$, or $p_t - v_t < 0$ and $\kappa_t = p_t - \theta_t > 0$, then $\partial |p_t - v_t| / \partial \kappa_t = -1$, which leads to $\partial m_t / \partial \kappa_t > 0$ conditional on Ω_3, Ω_4 or Ω_5. When chartists enhance mispricing, then $\partial |p_t - v_t| / \partial \kappa_t = 1$. In this case, $\partial m_t / \partial \kappa_t < 0$ if Ω_5 holds. Conditional on Ω_3 or Ω_4, $\partial m_t / \partial \kappa_t > 0$ if $\varepsilon_t < \sigma$, $\partial m_t / \partial \kappa_t < 0$ if $\varepsilon_t > \sigma$, and $\partial m_t / \partial \kappa_t = 0$ if $\varepsilon_t = \sigma$.

To summarize, $\partial m_t / \partial \kappa_t \leq 0$ if (i)$\partial |p_t - v_t| / \partial \kappa_t = 1$ and Ω_5, or (ii) $\partial |p_t - v_t| / \partial \kappa_t = 1$ and $\varepsilon_t \leq \sigma$ conditional on Ω_3 or Ω_4; otherwise, $\partial m_t / \partial \kappa_t \geq 0$. □

Proof of Lemma 3.2

Proof Differentiating m_t with respect to the information noise ε_t yields

$$\frac{\partial m_t}{\partial \varepsilon_t} = \begin{cases} 0 & \text{if } \Omega_1 \text{ or } \Omega_2 \\ \frac{\frac{\partial x_t^m}{\partial \varepsilon_t}\varepsilon_t - x_t^m + \theta_t}{2\varepsilon_t^2} & \text{if } \quad \Omega_3 \\ \frac{-\frac{\partial x_t^M}{\partial \varepsilon_t}\varepsilon_t - \theta_t + x_t^M}{2\varepsilon_t^2} & \text{if } \quad \Omega_4 \\ \frac{-\frac{\partial x_t^M}{\partial \varepsilon_t}\varepsilon_t + \frac{\partial x_t^m}{\partial \varepsilon_t}\varepsilon_t + x_t^M - x_t^m}{2\varepsilon_t^2} & \text{if } \quad \Omega_5 \end{cases}.$$

Recall the definition of x_t^m and x_t^M in Eqs. 11 and 5,

$$\frac{\partial x_t^m}{\partial \varepsilon_t} = \frac{-|p_t - v_t|}{\sigma}, \text{ and}$$
$$\frac{\partial x_t^M}{\partial \varepsilon_t} = \frac{|p_t - v_t|}{\sigma}.$$

So, $\partial m_t / \partial \varepsilon_t$ can be rewritten as

$$\frac{\partial m_t}{\partial \varepsilon_t} = \begin{cases} 0 & \text{if } \Omega_1, \Omega_2 \text{ or } \Omega_5 \\ \frac{\theta_t - p_t}{2\varepsilon_t^2} & \text{if } \quad \Omega_3 \\ \frac{p_t - \theta_t}{2\varepsilon_t^2} & \text{if } \quad \Omega_4 \end{cases}.$$

Given $\Omega_3 = \left\{x_t^m, x_t^M \mid x_t^m \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t] \text{ and } \theta_t + \varepsilon_t < x_t^M\right\}$, it must be true that $-\varepsilon_t |p_t - v_t| / \sigma + \varepsilon_t > \theta_t - p_t$, $-\varepsilon_t |p_t - v_t| / \sigma - \varepsilon_t < \theta_t - p_t$ and $\varepsilon_t |p_t - v_t| / \sigma - \varepsilon_t > \theta_t - p_t$. If $\varepsilon_t |p_t - v_t| / \sigma - \varepsilon_t > -\varepsilon_t |p_t - v_t| / \sigma + \varepsilon_t$, that is, $|p_t - v_t| > \sigma$, for Ω_3 to be true, it must be that $\theta_t - p_t < -\varepsilon_t |p_t - v_t| / \sigma + \varepsilon_t$, which suggests $\theta_t - p_t < 0$. If, on the other hand, $\varepsilon_t |p_t - v_t| / \sigma - \varepsilon_t \leq -\varepsilon_t |p_t - v_t| / \sigma + \varepsilon_t$, that is, $|p_t - v_t| \leq \sigma$, it must be true that $\theta_t - p_t < \varepsilon_t |p_t - v_t| / \sigma - \varepsilon_t \leq 0$.

Therefore, given Ω_3, $\theta_t - p_t < 0$ and therefore $\partial m/\partial \varepsilon_t < 0$. Similarly, we can show that $\theta_t - p_t > 0$ given $\Omega_4 = \left\{x_t^m, x_t^M \mid x_t^M \in [\theta_t - \varepsilon_t, \theta_t + \varepsilon_t] \text{ and } \theta_t - \varepsilon_t > x_t^m\right\}$, which leads to $\partial m_t/\partial \varepsilon_t < 0$. In conclusion, $\partial m_t/\partial \varepsilon_t \leq 0$, which suggests that smaller information noise or greater information transparency increases the fraction of fundamentalists in the market. □

Proof of Proposition 1

Proof Differentiating the price p_{t+1} in Eq. 17 with respect to the fundamental yields

$$\frac{\partial p_{t+1}}{\partial \theta_t} = \begin{cases} \dfrac{\gamma}{\alpha \varepsilon_t^2} & \text{if } \Omega_1 \text{ or } \Omega_5 \\ 0 & \text{if } \quad \Omega_2 \\ \dfrac{\gamma}{2\alpha\varepsilon_t}\left(\dfrac{p_t - \theta_t + \varepsilon_t}{\varepsilon_t^2} + \dfrac{p_t - v_t}{\beta\sigma^2}\right) & \text{if } \quad \Omega_3 \\ \dfrac{\gamma}{2\alpha\varepsilon_t}\left(\dfrac{\theta_t - p_t + \varepsilon_t}{\varepsilon_t^2} - \dfrac{p_t - v_t}{\beta\sigma^2}\right) & \text{if } \quad \Omega_4 \end{cases}.$$

Recall that $\theta_t < p_t$ given Ω_3, $\partial p_{t+1}/\partial \theta_t > 0$ if $p_t \geq v_t$. If $p_t < v_t$, then $\partial p_{t+1}/\partial \theta_t \leq 0$ if and only if $\frac{p_t - v_t}{\beta\sigma^2} < \frac{\theta_t - p_t - \varepsilon_t}{\varepsilon_t^2}$. As $x_t^m \in (\theta_t - \varepsilon_t, \theta_t + \varepsilon_t)$, $x_t^M > \theta_t + \varepsilon_t$ and $x_t^m = \varepsilon_t (p_t - v_t)/\sigma + p_t$ conditional on Ω_3, it follows that $\frac{p_t - v_t}{\beta\sigma^2} \in \left(\frac{\theta_t - p_t - \varepsilon_t}{\beta\sigma\varepsilon_t}, \frac{-|\theta_t - p_t + \varepsilon_t|}{\beta\sigma\varepsilon_t}\right)$ when $p_t < v_t$. Therefore, $\partial p_{t+1}/\partial \theta_t < 0$ if $\frac{p_t - v_t}{\beta\sigma^2}$ falls into $\left(\frac{\theta_t - p_t - \varepsilon_t}{\beta\sigma\varepsilon_t}, \min\left(\frac{-|\theta_t - p_t + \varepsilon_t|}{\beta\sigma\varepsilon_t}, \frac{\theta_t - p_t - \varepsilon_t}{\varepsilon_t^2}\right)\right)$, which is valid if and only if $\varepsilon_t > \beta\sigma$; otherwise $\partial p_{t+1}/\partial \theta_t \geq 0$. Clearly, it is impossible for $\partial p_{t+1}/\partial \theta_t < 0$ if $\varepsilon_t > \beta\sigma$. Recall the demand function of chartists in Eq. 8, given that $\kappa_t = |\theta_t - p_t|$, the condition for $\partial p_{t+1}/\partial \theta_t < 0$ is $y_t^c \in \left(\frac{-\kappa_t - \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{-|\theta_t - p_t + \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t - \varepsilon_t}{\alpha\varepsilon_t^2}\right)\right)$ and $\varepsilon_t > \beta\sigma$.

Similarly, we can show that, given Ω_4, $\partial p_{t+1}/\partial \theta_t < 0$ if $y^c \in \left(\max\left(\frac{|\theta_t - p_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t + \varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t + \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$ and $\varepsilon_t > \beta\sigma$; and $\partial p_{t+1}/\partial \theta_t \geq 0$ otherwise.

If $\kappa_t > \varepsilon_t$, then $-|\theta_t - p_t + \varepsilon_t| = \theta_t - p_t + \varepsilon_t = -(\kappa_t - \varepsilon_t)$ conditional on Ω_3, and $|\theta_t - p_t - \varepsilon_t| = \kappa_t - \varepsilon_t$ conditional on Ω_4. If $\kappa_t \leq \varepsilon_t$, then $-|\theta_t - p_t + \varepsilon_t| = (\kappa_t - \varepsilon_t)$ conditional on Ω_3, and $|\theta_t - p_t - \varepsilon_t| = -(\kappa_t - \varepsilon_t)$ conditional on Ω_4. So the condition for $\partial p_{t+1}/\partial \theta_t < 0$ can be summarized as $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t + \varepsilon_t}{\alpha\varepsilon_t^2}\right), \frac{\kappa_t + \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, $y_t^c(\theta_t - p_t) > 0$ and $\varepsilon_t > \beta\sigma$. □

Proof of Proposition 2

Proof Given Ω_3, $\theta_t < p_t$, $y_t^c \in \left\{\left(\frac{|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t + \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right) \cup \left(\frac{-\kappa_t - \varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{-|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right)\right\}$, and

$$\frac{\partial p_{t+1}}{\partial \theta_t} - \frac{\gamma}{\alpha\varepsilon_t^2} = \frac{\gamma}{2\alpha\varepsilon_t}\left(\frac{p_t - \theta_t - \varepsilon_t}{\varepsilon_t^2} + \frac{p_t - v_t}{\beta\sigma^2}\right).$$

So $\dfrac{\partial p_{t+1}}{\partial \theta_t} > \dfrac{\gamma}{\alpha \varepsilon_t^2}$ if (i) $y_t^c \in \left(\max\left(\frac{|\kappa_t - \varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2} \right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t} \right)$, which exists if and only if $\theta_t - p_t < \min\left(0, \frac{\varepsilon_t(\varepsilon_t - \beta\sigma)}{\beta\sigma+\varepsilon_t}\right)$; or (ii) $y_t^c \in \left(\max\left(\frac{-\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2} \right), \frac{-|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t} \right)$, which exists if and only if $\theta_t - p_t < -\varepsilon_t$ and $\varepsilon_t < \beta\sigma$.

Given Ω_4, $\theta_t - p_t > 0$, $y_t^c \in \left\{ \left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t} \right) \cup \left(\frac{-\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{-|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t} \right) \right\}$, and

$$\frac{\partial p_{t+1}}{\partial \theta_t} - \frac{\gamma}{\alpha\varepsilon_t^2} = \frac{\gamma}{2\alpha\varepsilon_t}\left(\frac{\theta_t - p_t - \varepsilon_t}{\varepsilon_t^2} - \frac{p_t - v_t}{\beta\sigma^2} \right).$$

So $\dfrac{\partial p_{t+1}}{\partial \theta_t} > \dfrac{\gamma}{A \varepsilon_t^2}$ if (i) $y_t^c \in \left(\frac{-\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{-|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2} \right) \right)$, which exists if and only if $\theta_t - p_t > \max(0, \frac{\varepsilon_t(\beta\sigma-\varepsilon_t)}{\beta\sigma+\varepsilon_t})$; or (ii) $y_t^c \in \left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2} \right) \right)$, which exists if and only if $\theta_t - p_t > \varepsilon_t$ and $\varepsilon_t < \beta\sigma$.

Clearly $\dfrac{\partial p_{t+1}}{\partial \theta_t} \le \dfrac{\gamma}{\alpha\varepsilon_t^2}$ conditional on Ω_1, Ω_2 and Ω_5.

To summarize, $\dfrac{\partial p_{t+1}}{\partial \theta_t} > \dfrac{\gamma}{\alpha\varepsilon_t^2}$ if and only if (i) $\left|y_t^c\right| \in \left(\max\left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \frac{-\kappa_t+\varepsilon_t}{\alpha\varepsilon_t^2} \right), \frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t} \right)$, $y_t^c(\theta_t - p_t) < 0$ and $\kappa_t > \max(0, \frac{\varepsilon_t(\beta\sigma-\varepsilon_t)}{\beta\sigma+\varepsilon_t})$, or (ii)$\left|y_t^c\right| \in \left(\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}, \min\left(\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}, \frac{\kappa_t-\varepsilon_t}{\alpha\varepsilon_t^2} \right) \right)$, $y_t^c(\theta_t - p_t) > 0$, $\kappa_t > \varepsilon_t$ and $\varepsilon_t < \beta\sigma$. □

Proof of Proposition 3

Proof Differentiating $\partial p_{t+1}/\partial\theta_t$ with respect to the degree of mispricing κ_t yields

$$\frac{\partial}{\partial \kappa_t}\frac{\partial p_{t+1}}{\partial \theta_t} = \begin{cases} 0 & \text{if } \Omega_1, \Omega_2 \text{ or } \Omega_5 \\ \dfrac{\gamma}{2\alpha\varepsilon_t}\left(\dfrac{1}{\varepsilon_t^2} + \dfrac{1}{\beta\sigma^2}\dfrac{\partial(p_t - v_t)}{\partial\kappa_t} \right) & \text{if } \quad \Omega_3 \\ \dfrac{\gamma}{2\alpha\varepsilon_t}\left(\dfrac{1}{\varepsilon_t^2} - \dfrac{1}{\beta\sigma^2}\dfrac{\partial(p_t - v_t)}{\partial\kappa_t} \right) & \text{if } \quad \Omega_4 \end{cases}.$$

Recall that $\kappa_t = p_t - \theta_t$ given Ω_3, $\partial(p_t - v_t)/\partial\kappa_t = 0$ if the change in κ_t is due to a change in θ_t, and $\dfrac{\partial(p_t - v_t)}{\partial\kappa_t} = 1$ if the change in κ_t is due to a change in p_t. Similarly, as $\theta_t > p_t$ conditional on Ω_4, $\partial(p_t - v_t)/\partial\kappa_t = 0$ if the change in κ_t is due to a change in θ_t. Otherwise, $\partial(p_t - v_t)/\partial\kappa_t = -1$. Therefore, $\dfrac{\partial}{\partial\kappa_t}\dfrac{\partial p_{t+1}}{\partial\theta_t} > 0$ conditional on Ω_3 or Ω_4, which suggests that the price is more responsive to a fundamental shock when the market becomes more inefficient (κ_t becomes larger). To summarize, we have $\dfrac{\partial}{\partial\kappa_t}\dfrac{\partial p_{t+1}}{\partial\theta_t} \ge 0$, where the equality holds if Ω_1, Ω_2 or Ω_5 is satisfied. □

Proof of Proposition 4

Proof Differentiating $\partial p_t/\partial\theta_t$ with respect to the information noise ε_t yields

$$\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t}=\begin{cases}-\dfrac{\gamma}{\alpha\varepsilon_t^3} & \text{if } \Omega_1 \text{ or } \Omega_5\\ 0 & \text{if } \quad \Omega_2\\ -\dfrac{\gamma}{2\alpha\varepsilon_t^2}\left[\dfrac{3\left(p_t-\theta_t\right)+2\varepsilon_t}{\varepsilon_t^2}+\dfrac{p_t-v_t}{\beta\sigma^2}\right] & \text{if } \quad \Omega_3\\ -\dfrac{\gamma}{2\alpha\varepsilon_t^2}\left[\dfrac{3\left(\theta_t-p_t\right)+2\varepsilon_t}{\varepsilon_t^2}-\dfrac{p_t-v_t}{\beta\sigma^2}\right] & \text{if } \quad \Omega_4\end{cases}.$$

Given Ω_3, $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t}>0$ if $y_t^c<\frac{3\left(\theta_t-p_t\right)-2\varepsilon_t}{\alpha\varepsilon_t^2}<0$. When $p_t<v_t$, $y_t^c\in\left(\frac{\theta_t-p_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t},\frac{-|\theta_t-p_t+\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right)$ conditional on Ω_3. Recall that $\theta_t<p_t$ conditional on Ω_3 and that $\kappa_t=|\theta_t-p_t|$, $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t}>0$ if and only if $y_t^c\in\left(\frac{-\kappa_t-\varepsilon_t}{\alpha\beta\sigma\varepsilon_t},\min\left(\frac{-3\kappa_t-2\varepsilon_t}{\alpha\varepsilon_t^2},\frac{-|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right)\right)$, which exists if $\frac{-3\kappa_t+\varepsilon_t}{\varepsilon_t^2}>\frac{-\kappa_t-\varepsilon_t}{\beta\sigma\varepsilon_t}$, that is (i) $\varepsilon_t\geq 3\beta\sigma$, or (ii) $\varepsilon_t\in(2\beta\sigma,3\beta\sigma)$ and $\kappa_t<\frac{(\varepsilon_t-2\beta\sigma)\varepsilon_t}{3\beta\sigma-\varepsilon_t}$.

Similarly, given Ω_4, $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t}>0$ if $y_t^c\in\left[\max\left(\frac{3\kappa_t+2\varepsilon_t}{\alpha\varepsilon_t^2},\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right),\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right]$, which exists if (i) $\varepsilon_t\geq 3\beta\sigma$, or (ii) $\varepsilon_t\in(2\beta\sigma,3\beta\sigma)$ and $\kappa_t<\frac{(\varepsilon_t-2\beta\sigma)\varepsilon_t}{3\beta\sigma-\varepsilon_t}$.

To summarize, $\frac{\partial}{\partial\varepsilon_t}\frac{\partial p_{t+1}}{\partial\theta_t}\geq 0$ if and only if $\left|y_t^c\right|\in\left(\max\left(\frac{3\kappa_t+2\varepsilon_t}{\alpha\varepsilon_t^2},\frac{|\kappa_t-\varepsilon_t|}{\alpha\beta\sigma\varepsilon_t}\right),\frac{\kappa_t+\varepsilon_t}{\alpha\beta\sigma\varepsilon_t}\right)$, $y_t^c\left(\theta_t-p_t\right)>0$ and either (i) $\varepsilon_t\geq 3\beta\sigma$, or (ii) $\varepsilon_t\in(2\beta\sigma,3\beta\sigma)$ and $\kappa_t<\frac{(\varepsilon_t-2\beta\sigma)\varepsilon_t}{3\beta\sigma-\varepsilon_t}$. □

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